Extracting and using photon polarization information in radiative B decays

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Abstract

We discuss the uses of conversion electron pairs for extracting photon polarization information in weak radiative B decays. Both cases of leptons produced through a virtual and real photon are considered. Measurements of the angular correlation between the $(K\pi)$ and (e^+e^-) decay planes in $B \to K^*(\to K\pi)\gamma^{(*)}(\to e^+e^-)$ decays can be used to determine the helicity amplitudes in the radiative $B \to K^*\gamma$ decay. A large right-handed helicity amplitude in \bar{B} decays is a signal of new physics. The time-dependent CP asymmetry in the B^0 decay angular correlation is shown to measure $\sin 2\beta$ and $\cos 2\beta$ with little hadronic uncertainty.

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I. INTRODUCTION

Rare decays of B mesons have attracted a lot of attention due to their ability to probe the existence of new physics. The most accessible such processes are weak radiative decays mediated by the quark process $b \to s(d)\gamma$. The CLEO Collaboration reported measurements of both exclusive and inclusive branching ratios for the $b \to s\gamma$ process, with results in good agreement with the Standard Model (SM) predictions [1–3]. This still leaves open the possibility that new physics could be present, but it manifests itself only in the details of the decay process, such as polarization effects or differential distributions. A number of methods have been proposed which could detect deviations from the SM predictions along these lines [4–10].

One particular class of methods is based on the SM prediction that the photons emitted in $b \to s\gamma$ decays are predominantly left-handed. (Long-distance effects and the light-quark masses introduce a small right-handed component, which can be neglected to a first approximation.) This property does not hold true in extensions of the SM such as the left-right symmetric model (LRM) [11], and therefore, can be used to signal the presence of new physics. Unfortunately, in B decays all photon polarization information contained in the final hadron is lost. Since the photon polarization in $\bar{B} \to K^* \gamma$ decays is difficult to detect, most polarization-based methods focused on the related $b \to s \ell^+ \ell^-$ decay, where the angular distributions and lepton polarizations can probe the chiral structure of the short-distance matrix element [5,6,8–10].

Detecting an unambiguous signal for new physics entails a good control over the SM prediction for the respective exclusive modes. While the short-distance contribution can be parametrized in terms of (more or less well-known) hadronic form-factors, the charmand up-quark loops introduce hard-to-calculate long-distance contributions. In $b \to s\ell^+\ell^-$ decays, these effects become significant when q^2 (the e^+e^- invariant mass) is in the region of the charmonium excitations, and are minimal at the lower end $(q^2 \to 0)$. Furthermore, the differential rate is enhanced at this point due to the small photon propagator denominator. The remaining long-distance contributions, connected with weak annihilation or W-exchange topologies, can be computed in an expansion in $1/q_0$ [12]. This suggests that the $b \to s\gamma$ decay, respectively the $b \to s\ell^+\ell^-$ decay in the small- q^2 region, is a good place to search for new physics through photon polarization effects.

Measuring mixing-induced CP asymmetries in the inclusive $b \to s \gamma$ decay was proposed as an indirect method for probing photon polarization effects in [4]. Since both B and \bar{B} must decay to a common final state, the resulting asymmetry measures the interference of right- and left-handed photon amplitudes. As the SM predicts a very small right-handed admixture of photons in $b \to q \gamma$ decays, a large mixing-induced CP asymmetry is a signal of new physics.

We explore in this paper an alternative way of measuring the photon polarization in the exclusive $B \to V \gamma$ decay, which makes use of the conversion electron pairs formed by the primary photon. Electron-positron pairs from photons that were produced in the inner part of the detector can be traced and their decay plane can be reconstructed with high accuracy. For example, at BaBar about 3% of the photons are expected to convert on the beam pipe and the silicon detector [13]. We show in Sec. II how the conversion process can be used to extract information about the photon polarization in $B \to V \gamma$ decays,

in analogy to the classical example of determining the π^0 parity through the chain decay $\pi^0 \to \gamma(e^+e^-)\gamma(e^+e^-)$ [14]. The main ingredient of this analysis is the fact that in B decay into two vectors, the subsequent decay (or conversion) of the vectors contains their relative polarization information. In particular, the angular distribution in the relative angle of the $K^* \to K\pi$ decay plane and that of the conversion pair can be used to determine the helicity amplitudes in the $B \to V\gamma$ decay. In Sec. III we discuss time-dependent CP asymmetries in the angular distribution of the conversion pairs produced in neutral B decays, which measure $\sin 2\beta$ and $\cos 2\beta$ with little hadronic uncertainty. We comment in Sec. IV on the experimental aspects of the methods proposed here.

II. CONVERSION LEPTON PAIRS

There are two possible mechanisms by which the photon (real or virtual) emitted in $b \to s\gamma$ decay can convert into a lepton pair. In the first one, a virtual photon with momentum $q^2 \geq (2m_e)^2$ produces an electron-positron pair, which are emitted without any other interaction. In the second mechanism, a real photon produces a lepton pair which subsequently interacts with a nucleus by Coulomb forces (the so-called Bethe-Heitler process [15]).

The lepton pair produced in these two processes are seen very differently in a detector. The first mechanism produces prompt lepton pairs, which originate practically from the interaction region. The pairs would be produced even in vacuum, in the absence of any matter content of the detector. On the other hand, the Bethe-Heitler process produces lepton pairs within the volume of the detector with a probability proportional to the density of matter.

These arguments can be illustrated with a simple estimate as follows. The lifetime of a virtual photon contributing to the first mechanism is (in its rest frame) of the order $\tau_0 \simeq 1/\sqrt{q^2} \sim O(1/m_e)$. In the lab frame the lifetime is longer by a factor of $\gamma = q_0/\sqrt{q^2} \sim O(m_B/m_e)$. Thus the photon travels a distance $\Delta x \sim m_B/m_e^2 \sim 10^{-6}~mm$ before decaying. Clearly, any photon that has $q^2 > (2m_e)^2$ travels a distance that is too short to be measured. For this reason we will refer to the lepton pairs produced in these two mechanisms as to short-distance and long-distance conversion leptons, respectively. In the following we examine them in turn.

A. Short-distance lepton pairs

In the Standard Model, the decays $\bar{B} \to X_s e^+ e^-$ are mediated by a combination of the short-distance Hamiltonian [16–18]

$$\mathcal{H}_{\text{s.d.}} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=7-10} C_i(\mu) \mathcal{O}_i(\mu)$$
 (2.1)

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$$\mathcal{O}_{7} = \frac{e}{16\pi^{2}} \bar{s} \sigma_{\mu\nu} (m_{b} P_{R} + m_{s} P_{L}) b F^{\mu\nu} , \quad \mathcal{O}_{8} = \frac{g}{16\pi^{2}} \bar{s} \sigma_{\mu\nu} (m_{b} P_{R} + m_{s} P_{L}) T^{a} b G^{a\mu\nu} \qquad (2.2)$$

$$\mathcal{O}_{9} = \frac{e^{2}}{16\pi^{2}} (\bar{s} \gamma_{\mu} P_{L} b) (\bar{e} \gamma^{\mu} e) , \qquad \mathcal{O}_{10} = \frac{e^{2}}{16\pi^{2}} (\bar{s} \gamma_{\mu} P_{L} b) (\bar{e} \gamma^{\mu} \gamma_{5} e)$$

and nonlocal contributions introduced by the usual weak nonleptonic Hamiltonian

$$\mathcal{H}_{W} = \frac{4G_{F}}{\sqrt{2}} \left\{ V_{ub} V_{us}^{*} \left[C_{1} \mathcal{O}_{1}^{(u)} + C_{2} \mathcal{O}_{2}^{(u)} \right] + V_{cb} V_{cs}^{*} \left[C_{1} \mathcal{O}_{1}^{(c)} + C_{2} \mathcal{O}_{2}^{(c)} \right] - V_{tb} V_{ts}^{*} \sum_{i=3}^{10} C_{i}(\mu) \mathcal{O}_{i}^{(s)} + (s \to d) \right\}$$

$$(2.3)$$

with

$$\mathcal{O}_1^{(q)} = (\bar{q}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L q), \quad \mathcal{O}_2^{(q)} = (\bar{s}\gamma_\mu P_L b)(\bar{q}\gamma^\mu P_L q). \tag{2.4}$$

With new physics other operators can also contribute. In particular, the right handed operators, which can be obtained from the SM operators by $R \leftrightarrow L$, are denoted by $\tilde{\mathcal{O}}_i$.

The amplitude for $\bar{B} \to V e^+ e^-$ decay, with V a light vector meson, can be written as

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* e \left\{ (A_\mu + \frac{1}{q^2} H_\mu) [\bar{u}(p_{e_+}) \gamma^\mu v(p_{e^-})] + B_\mu [\bar{u}(p_{e_+}) \gamma^\mu \gamma_5 v(p_{e^-})] \right\}, \qquad (2.5)$$

with A, B, H hadronic matrix elements. B_{μ} receives only contributions from the operator \mathcal{O}_{10} above, and can be expressed in terms of form-factors alone. A and H can receive contributions from all the other operators. The part which contains a pole at $q^2 = 0$, arising from the propagator of an intermediate photon, dominates in the small- q^2 region, in which we will be interested in the following. Therefore, we will neglect the nonpole terms proportional to A_{μ} and B_{μ} .

The matrix element H_{μ} is parametrized in the most general form in terms of three invariant form factors

$$H_{\mu}(q^2) = A_{\parallel}(q^2) \left(q_0 \epsilon_{V\mu}^* - v_{\mu}(q \cdot \epsilon_V^*) \right) + A_0(q^2) v_{\mu}(q \cdot \epsilon_V^*) + A_{\perp}(q^2) i \varepsilon(q, \mu, \epsilon_V^*, v) . \tag{2.6}$$

The amplitudes $A_{\parallel}(q^2)$ (CP-even) and $A_{\perp}(q^2)$ (CP-odd) correspond to the virtual photon polarization being parallel, respectively transverse to that of the vector meson. $A_0(q^2)$ (CP-even) is related to the longitudinal polarization. The values of these form factors at $q^2 = 0$ describe the coupling of a real photon in the weak radiative decay $\bar{B} \to V\gamma$. Thus, in general, $A_0(0) = 0$ and in the SM, where the photon is mostly left handed, we have $A_{\parallel}(0) \simeq -A_{\perp}(0)$.

An alternative description of the photon coupling is in terms of helicity amplitudes, giving the amplitude for the \bar{B} meson to decay into a virtual photon of well-defined helicity. They are related to the invariant form factors in (2.6) by

$$A_{\ell}(q^2) = \sqrt{q^2 \frac{m_B - q_0}{m_V}} A_{\parallel}(q^2) + \frac{\vec{q}^2 m_B}{m_V \sqrt{q^2}} A_0(q^2)$$
 (2.7)

$$A_{R,L}(q^2) = -q_0 A_{\parallel}(q^2) \mp |\vec{q}| A_{\perp}(q^2).$$
(2.8)

Here A_R is the right (left) handed polarization amplitude and A_ℓ is the longitudinal polarization amplitude which clearly vanish in the $q^2=0$ limit. Expressed in terms of helicity amplitudes, the rate for radiative decay $\bar{B} \to V \gamma$ is given by

$$\Gamma(\bar{B} \to V\gamma) = \frac{G_F^2 |V_{ts}V_{tb}^*|^2}{\pi m_B^2} E_{\gamma}(|A_R(0)|^2 + |A_L(0)|^2), \qquad (2.9)$$

where E_{γ} is the photon energy in the \bar{B} rest frame.

In the SM, the $\bar{B} \to V \gamma$ amplitude is dominated by the operator \mathcal{O}_7 , which contributes mainly to A_L , with a small right-handed admixture due to long-distance effects and light quark masses. In certain extensions of the SM, such as the left-right symmetric model [11], a new penguin operator $\tilde{\mathcal{O}}_7$ is introduced involving right-handed photons. This operator can make a significant contribution to A_R . Thus, the photon amplitudes are given, in the general case, by

$$A_R(0) = -(\tilde{C}_7 m_b + C_7 m_q) \frac{e}{16\pi^2} 2(m_B^2 - m_V^2) g_+(0) + a_R,$$

$$A_L(0) = (C_7 m_b + \tilde{C}_7 m_q) \frac{e}{16\pi^2} 2(m_B^2 - m_V^2) g_+(0) + a_L.$$
(2.10)

where the form factor $g_+(q^2)$ is defined by

$$\langle V(p')|\bar{q}\sigma_{\mu\nu}b|B(p)\rangle = g_{+}(q^{2})\varepsilon_{\mu\nu\lambda\sigma}\epsilon_{\lambda}^{*}(p+p')_{\sigma} + g_{-}(q^{2})\varepsilon_{\mu\nu\lambda\sigma}\epsilon_{\lambda}^{*}(p-p')_{\sigma}$$

$$+h(q^{2})\varepsilon_{\mu\nu\lambda\sigma}(p+p')_{\lambda}(p-p')_{\sigma}(\epsilon^{*}\cdot p).$$

$$(2.11)$$

 m_q is the mass of the strange or down quark depending on the specific decay. The long-distance amplitudes $a_{L,R}$ are introduced by the $b \to c\bar{c}s$ part of the weak Hamiltonian and are expected to be about 5% of the short-distance contribution [19–21,12].

We emphasize here that in the SM the photon in $B \to V \gamma$ is almost pure left-handed. The small right handed component due to long distance effects and light quark masses is at the 5% level. Thus, any measurement of a significant right handed amplitude will be an unambiguous signal for new physics.

A measurement of the individual helicity amplitudes $A_{L,R}(0)$ can therefore give useful information about the short-distance weak radiative Hamiltonian. We will show in the following how a study of the decay $\bar{B} \to V e^+ e^-$ in the low $e^+ e^-$ invariant mass region can be useful in this respect. The argument is a simple adaptation of the classical analysis of Kroll and Wada given in [22].

Let us take the virtual photon to be moving along the +z axis, and the final state meson V in the -z direction, with momenta \vec{q} and $-\vec{q}$ respectively. The photon converts into a e^+e^- pair, with the e^+ moving at an azimuthal angle θ with respect to the +z axis and a polar angle ϕ . The vector meson decays into two pseudoscalars, which will be denoted generically by $K(p_K)$ and $\pi(p_{\pi})$ (corresponding to the interesting case $V = K^*$). The pion momentum p_{π} is parametrized by the angles $(\psi, 0)$ with respect to the -z direction. The differential decay rate in these coordinates is given by

$$\frac{d\Gamma}{dq^{2}d\cos\theta d\cos\psi d\phi} = \mathcal{C}\left\{4|A_{\ell}|^{2}\sin^{2}\theta\cos^{2}\psi + (|A_{R}|^{2} + |A_{L}|^{2})(1 + \cos^{2}\theta)\sin^{2}\psi \right. (2.12)$$

$$-\sin 2\theta\sin 2\psi\left\{\cos\phi\left[\operatorname{Re}(A_{\ell}A_{R}^{*}) + \operatorname{Re}(A_{\ell}A_{L}^{*})\right] + \sin\phi\left[\operatorname{Im}(A_{\ell}A_{R}^{*}) - \operatorname{Im}(A_{\ell}A_{L}^{*})\right]\right\}$$

$$-2\left[\operatorname{Re}(A_{R}A_{L}^{*})\cos 2\phi - \operatorname{Im}(A_{R}A_{L}^{*})\sin 2\phi\right]\sin^{2}\theta\sin^{2}\psi\right\}.$$

The constant \mathcal{C} is given by

$$C = \frac{3\alpha G_F^2 |V_{tb}V_{ts}^*|^2}{8(2\pi)^3 m_B^3} \cdot \frac{\sqrt{\lambda}}{q^2}, \qquad \lambda = \frac{1}{4} (m_B^2 - q^2 - m_V^2)^2 - q^2 m_V^2.$$
 (2.13)

We assumed in deriving (2.12) that the final leptons are massless, and neglected the parity-violating effects in the decay $\gamma^* \to e^+e^-$. Such effects are introduced by Z boson exchange (the operator \mathcal{O}_{10}), and are parametrized by the hadronic matrix element B_{μ} in (2.5). They are negligibly small in the small q^2 region we consider here. The form factor $A_0(q^2)$ vanishes at $q^2 = 0$ as $A_0(q^2) \propto q^2$, such that the amplitude for producing longitudinally polarized real photons $A_{\ell}(q^2)$ vanishes for $q^2 = 0$, as expected.

From (2.12) one obtains, after integrating over (θ, ψ) , the following ϕ distribution

$$\frac{d\Gamma}{d\phi} = \frac{32}{9} \int_{(2m_e)^2}^{q_{\text{max}}^2} dq^2 \mathcal{C}(q^2) \left\{ (|A_\ell(q^2)|^2 + |A_R(q^2)|^2 + |A_L(q^2)|^2) - \left[\text{Re}(A_R(q^2) A_L^*(q^2)) \cos 2\phi - \text{Im}(A_R(q^2) A_L^*(q^2)) \sin 2\phi \right] \right\}.$$
(2.14)

In the region close to threshold, the helicity amplitude for producing longitudinally polarized photons has the asymptotic form $|A_{\ell}(q^2)|^2 \propto q^2$. Furthermore, to a first approximation, one can neglect the q^2 -variation of the transverse helicity amplitudes $|A_{R,L}(q^2)|$ in this region and set them equal to their values at $q^2 = 0$. Therefore, the q^2 -integral can be approximated as

$$\frac{d\Gamma}{d\phi} = \frac{1}{2\pi} \Gamma(\bar{B} \to V\gamma) \left(\frac{\alpha}{3\pi} \log \frac{q_{\text{max}}^2}{(2m_e)^2} \right)
\times \left\{ 1 - \frac{\text{Re}(A_R(0)A_L^*(0))\cos 2\phi - \text{Im}(A_R(0)A_L^*(0))\sin 2\phi}{|A_R(0)|^2 + |A_L(0)|^2} \right\} + \cdots$$
(2.15)

The ellipsis denote a neglected contribution from the longitudinal amplitude, proportional to the integral $\int dq^2/q^2 |A_\ell(q^2)|^2$. At $q_{\rm max}^2 = 1~{\rm GeV^2}$, it amounts to about 2% of the leading terms which are kept. Note the presence of the large logarithm $\log \frac{q_{\rm max}^2}{(2m_e)^2}$, which can partly compensate the additional suppression of α compared to the purely radiative rate (2.9). Numerically, the value of the suppression factor in brackets is $\frac{\alpha}{3\pi} \log \frac{q_{\rm max}^2}{(2m_e)^2} \simeq 0.01$ at $q_{\rm max}^2 = 1$ GeV². This implies an effective branching ratio of few times 10^{-7} for events with q^2 in the region of interest.

From (2.15) one can see that from the ϕ dependence in the q^2 -integrated rate we can extract the ratio

$$R \equiv \frac{|A_R(0)| |A_L(0)|}{|A_R(0)|^2 + |A_L(0)|^2}.$$
 (2.16)

Combining this with the total exclusive rate (2.9), the individual helicity amplitudes for right- and left-handed photons can be extracted (up to a $A_R(0) \leftrightarrow A_L(0)$ ambiguity). In the SM we expect $R \leq 5\%$. Therefore, by measuring R we are sensitive to new physics amplitudes that are an order of magnitude smaller than the SM amplitude.

Angular distributions of the type (2.15) in the low dilepton invariant mass region were also discussed in [5,10]. There, the full expressions for the helicity amplitudes are kept, including parity-violating effects induced by the operator \mathcal{O}_{10} . The resulting form of the angular distribution depends on many form-factors and is therefore not easily connected to the parameters of the short-distance Hamiltonian. In contrast, the phenomenological analysis presented here is model-independent; by restricting to a sufficiently small region above $q^2 = 0$, the radiative helicity amplitudes can be directly extracted, without need for any additional form factors.

B. Long-distance lepton pairs

Angular correlations in lepton pair production by a real photon have been suggested long ago as a means for measuring photon polarization (for a review see [23]). Our discussion here will focus on aspects relevant to the photons emitted in exclusive radiative B decays.

The cross-section for pair production by a polarized photon was computed in [24–26]. To lowest order it is given by

$$\frac{d\sigma}{dE_{1}d\Omega_{1}d\Omega_{2}} = \frac{Z^{2}e^{6}}{16\pi^{3}} \frac{|\vec{p}_{1}||\vec{p}_{2}|}{E_{\gamma}^{3}k^{4}} \left\{ \frac{(k^{2} - 4E_{2}^{2})(\vec{e} \cdot \vec{p}_{1})(\vec{e}^{*} \cdot \vec{p}_{1})}{(E_{1} - |\vec{p}_{1}|\cos\theta_{1})^{2}} + \frac{(k^{2} - 4E_{1}^{2})(\vec{e} \cdot \vec{p}_{2})(\vec{e}^{*} \cdot \vec{p}_{2})}{(E_{2} - |\vec{p}_{2}|\cos\theta_{2})^{2}} \right. \\
\left. - \frac{k^{2} + 4E_{1}E_{2}}{(E_{1} - |\vec{p}_{1}|\cos\theta_{1})(E_{2} - |\vec{p}_{2}|\cos\theta_{2})} [(\vec{e} \cdot \vec{p}_{1})(\vec{e}^{*} \cdot \vec{p}_{2}) + (\vec{e}^{*} \cdot \vec{p}_{1})(\vec{e} \cdot \vec{p}_{2})] \right. \\
\left. + E_{\gamma}^{2} \frac{\vec{p}_{1}^{2} \sin^{2}\theta_{1} + \vec{p}_{2}^{2} \sin^{2}\theta_{2} + 2|\vec{p}_{1}||\vec{p}_{2}|\sin\theta_{1}\sin\theta_{2}\cos(\phi_{1} - \phi_{2})}{(E_{1} - |\vec{p}_{1}|\cos\theta_{1})(E_{2} - |\vec{p}_{2}|\cos\theta_{2})} \right\}. \tag{2.17}$$

We denote here with \vec{e} the photon polarization vector and $p_1(p_2)$ the positron (electron) momenta. $k = p_1 + p_2 - q$ is the momentum transferred to the nucleus, which will be taken to be infinitely heavy. In this limit the nucleus does not carry away any energy and the photon energy is transferred entirely to the electron pair $E_{\gamma} = E_1 + E_2$. The angles of the positron and electron with respect to the photon momentum direction are denoted with θ_1 and θ_2 respectively.

We propose to use as polarization analyzer the rate (2.17) integrated over the electron direction Ω_2 . For a linearly polarized photon, the integrated cross-section for pair production has the following dependence on the angle α between the photon polarization vector and the projection of the positron momentum \vec{p}_1 on the plane transverse to the photon momentum

$$\frac{d\sigma}{dE_1 d\Omega_1} = \sigma_I + \frac{1}{2} (\sigma_{II} - \sigma_{III}) \cos 2\alpha + \sigma_{IV} \sin 2\alpha.$$
 (2.18)

We used here the notations of [26] for the pair production cross-section by a polarized photon. Only the cross-sections σ_I , σ_{II} and σ_{III} have been computed in this paper (see Eqs. (17) in [26]) corresponding to unpolarized photons (σ_I), and linearly polarized photons with the (\vec{p}_1 , \vec{q}) plane parallel to the polarization plane (σ_{II}) and transverse to it (σ_{III}). One has $\sigma_I = \frac{1}{2}(\sigma_{II} + \sigma_{III})$. The cross-section σ_{IV} measures the acoplanarity of the three vectors \vec{q} , \vec{p}_1 , \vec{p}_2 . The lepton pair is predominantly produced such that the photon momentum \vec{q} lies in the plane of the pair. Therefore, σ_{IV} is very small and will be neglected in the numerical estimates below.

Once the vector meson in $B \to V \gamma$ is observed through its decay to a pair of pseudoscalars (whose decay plane defines the x axis), the polarization vector of the emitted photon is fixed to be $\vec{e} \propto A_{\parallel} \vec{e}_1 + i A_{\perp} \vec{e}_2$. The angular distribution of the positron momentum direction is then

$$\frac{d\sigma}{dE_{1}d\Omega_{1}} = \Gamma(B \to V\gamma)\sigma_{I} \left\{ 1 + \cos 2\phi \left[\frac{\sigma_{II} - \sigma_{III}}{\sigma_{I}} \cdot \frac{\text{Re}(A_{R}A_{L}^{*})}{|A_{R}|^{2} + |A_{L}|^{2}} + \frac{2\sigma_{IV}}{\sigma_{I}} \cdot \frac{\text{Im}(A_{R}A_{L}^{*})}{|A_{R}|^{2} + |A_{L}|^{2}} \right] + \sin 2\phi \left[-\frac{\sigma_{II} - \sigma_{III}}{\sigma_{I}} \cdot \frac{\text{Im}(A_{R}A_{L}^{*})}{|A_{R}|^{2} + |A_{L}|^{2}} + \frac{2\sigma_{IV}}{\sigma_{I}} \cdot \frac{\text{Re}(A_{R}A_{L}^{*})}{|A_{R}|^{2} + |A_{L}|^{2}} \right] \right\}, \quad (2.19)$$

where the angle ϕ defined before Eq. (2.12). The dependence on ϕ has the form

$$\frac{d\sigma}{d\phi} \propto 1 + \xi R \cos(2\phi + \delta) \,, \tag{2.20}$$

with R the ratio of amplitudes defined in (2.16), and ξ an efficiency factor which can be expressed in terms of the cross-sections σ_{I-IV} as

$$\xi(E_1, \theta_1) = \sqrt{\left(\frac{\sigma_{II} - \sigma_{III}}{\sigma_I}\right)^2 + 4\left(\frac{\sigma_{IV}}{\sigma_I}\right)^2}, \quad \xi \le 2.$$
 (2.21)

Most of the pairs produced in the Bethe-Heitler process are emitted in a small cone of opening angle 2θ with $\theta \simeq m/E_{\gamma} \simeq 0.01^{\circ}$. We plot in Fig. 1 the efficiency parameter $\xi(E_1, \theta_1)$ as a function of θ_1 in this range, for three values of the positron energy E_1 . The sensitivity to the photon polarization is smaller by about a factor of 2 than for the short-distance lepton pairs (see Eq. (2.15)), and is maximal when the photon energy is equally distributed among the two leptons $(E_1 = E_{\gamma}/2)$.

In Fig. 1 we show also the effective ξ parameter obtained when the positron energy is not measured. This is defined as in (2.21) but in terms of the cross-sections integrated over E_1 . Averaging over E_1 further decreases the sensitivity to the photon polarization. An alternative method has been discussed in the literature [27] which improves the sensitivity at the cost of statistics. This method uses as polarization analyzer the rate (2.17) integrated over a restricted region of the electron direction Ω_2 , chosen such that the three vectors $\vec{q}, \vec{p_1}, \vec{p_2}$ are almost coplanar ($|\phi_2 - \phi_1 - \pi| \leq \Delta \phi$). A detailed discussion of the resulting asymmetry as a function of the width $\Delta \phi$ can be found in [27]. However, the gain in sensitivity of this method may be offset by a loss in statistics involved by integrating over a restricted region in Ω_2 .

To conclude this section we stress again the main point. In the SM, as R is very small, there is almost no angular dependence in the electron-positron conversion rate. Any significant measurement of such angular dependence will be an indication of new physics. In principle, if indeed such new physics exists, using the formulae presented in this section one could also determine the relative size of this new physics amplitude. While this may be hard to achieve, the modest goal of demonstrating any angular distribution may be experimentally feasible if $R \sim O(1)$.

III. TIME-DEPENDENT ANGULAR DISTRIBUTIONS

A different aspect of polarization effects in weak radiative decays is manifested through time-dependence in neutral B decays. Assuming the validity of the SM, we will show that certain time-dependent CP asymmetries involving real or virtual photons can be used to measure $\sin 2\beta$ and $\cos 2\beta$ with very little hadronic uncertainty. While the measurement of the polarization is sensitive to the right-handed operator $\tilde{\mathcal{O}}_7$, the CP asymmetry is sensitive to a new CP violating amplitude independent of its helicity. In the presence of any new contribution to the decay amplitude with a weak phase that is different from the SM phase, the "would be" $\sin 2\beta$ measured in the radiative decay would not agree with the one measured in $B \to \psi K_S$. Such a disagreement will be a clean signal for a new CP violating amplitude

in $b \to s\gamma$ [28]. In addition, the sign of $\cos 2\beta$ can be used to resolve a discrete ambiguity in the value of 2β deduced from the measurement of $\sin 2\beta$ [29]. Therefore, this measurement is sensitive to new CP violating contributions to the mixing amplitude.

Before developing the formalism we explain below why we gain sensitivity to CP violation phases by using polarization information. Atwood et~al.~[4] studied the time-dependent CP asymmetries in $B^0(t) \to V \gamma$ where no polarization information is obtained. They conclude that in the SM the asymmetry almost vanishes and only in the presence of right-handed amplitude there is going to be an asymmetry. The reason for this is simple. In the SM, \bar{B}^0 decays into a left-handed photon, while B^0 decays into a right-handed photon. However, interference is necessary in order to produce an asymmetry. Therefore, a final state that is accessible only from B^0 or \bar{B}^0 does not produce any asymmetry. In contrast, when we consider decays into a linear polarized photon, both B^0 and \bar{B}^0 can decay into the same final state. This is because the linear polarization state contains an equal mixture of the circular polarization states. Moreover, unless there are at least two amplitudes with different weak and strong phases, the magnitudes of the decay amplitudes into a linear polarized state for both B^0 or \bar{B}^0 are the same.

The situation is very similar to the well known $B^0 \to \psi K$ decay. The B^0 decays only to ψK^0 while the \bar{B}^0 decays into $\psi \bar{K}^0$ and there is no interference between the two decays. Indeed, if we measure $B \to \psi K$ without determining any property of the final kaon, we do not get any asymmetry. The situation is very different when we look into final state kaons that are admixtures of K and \bar{K} , namely K_S and K_L . In that case, both B^0 and \bar{B}^0 decay to the same final state and the asymmetry can be used to measure $\sin 2\beta$ in the SM. In both $B \to V \gamma$ and $B \to \psi K_S$ cases the situation is the same: in order to measure the asymmetry we have to observe final states that are accessible from both B^0 and \bar{B}^0 .

At this point we already know what can be measured in the CP asymmetry in $B \to K^* \gamma$ with linear polarized photon. Since these final states are CP eigenstates we can use the well known formalism of CP asymmetries in B decays into CP eigenstates. In the SM (working in the Wolfenstein parametrization) the $b \to s\gamma$ amplitude has a trivial weak phase, and thus the asymmetry is sensitive to the mixing amplitude, namely to 2β . Moreover, since we have many amplitudes that interfere we are sensitive to both $\sin 2\beta$ and $\cos 2\beta$. Below we show how to extract $\sin 2\beta$ and $\cos 2\beta$ from the angular distribution information.

In $B^0 \to V \gamma$ decays, final states of well-defined CP correspond to the amplitudes $A_{\parallel}(0)$ and $A_{\perp}(0)$, rather than to states of well-defined helicity. Furthermore, if $V = K^{*0}, \overline{K^{*0}}$, one must require that the final state be identified through the decay $K^{*0} \to K_S \pi^0$, which is CP-odd. We denote the corresponding amplitudes in $\overline{B}^0 \to V \gamma$ decays by \overline{A}_{\parallel} and $\overline{A}_{\perp}^{-1}$.

If the decaying meson is tagged as $B^0(\bar{B}^0)$ at time t=0, then the amplitudes A_{\parallel} and A_{\perp} will depend on t at a later time t. This time dependence is given by

$$A_{\parallel}(t) = A_{\parallel}(0) \left(f_{+}(t) + \lambda_{\parallel} f_{-}(t) \right)$$
(3.1)

$$\bar{A}_{\parallel}(t) = \frac{p}{q} A_{\parallel}(0) \left(f_{-}(t) + \lambda_{\parallel} f_{+}(t) \right) , \qquad (3.2)$$

¹Note the change in notation compared to Sec. II. To conform with usual conventions, $B(\bar{B})$ decay amplitudes will be denoted with $A(\bar{A})$, whereas in Sec. II we dealt only with \bar{B} decay amplitudes.

and analogous for A_{\perp} with a different parameter λ_{\perp} . Our notation for the $B - \bar{B}$ mixing parameters p, q is the standard one [30]. The time-dependence is contained in the functions

$$f_{\pm}(t) = \frac{1}{2} \left\{ e^{-i(m_1 - i\Gamma_1/2)t} \pm e^{-i(m_2 - i\Gamma_2/2)t} \right\}, \qquad (3.3)$$

with $m_{1,2}$ and $\Gamma_{1,2}$ the masses and widths of the mass eigenstates of the $B-\bar{B}$ system. The parameters λ_{\parallel} and λ_{\perp} are defined as usual by

$$\lambda_{\parallel} = \frac{q}{p} \frac{\bar{A}_{\parallel}(0)}{A_{\parallel}(0)}, \qquad \lambda_{\perp} = \frac{q}{p} \frac{\bar{A}_{\perp}(0)}{A_{\perp}(0)}.$$
 (3.4)

In the following we concentrate on B^0 decay via the $b \to s\gamma$ quark level transition. Within the SM, the dominance of the left-handed amplitude implies

$$A_{\parallel} \simeq A_{\perp} \,, \qquad \bar{A}_{\parallel} \simeq -\bar{A}_{\perp} \,, \tag{3.5}$$

where we used $A_L = -A_{\parallel} + A_{\perp} \approx 0$ and $\bar{A}_R = -\bar{A}_{\parallel} - \bar{A}_{\perp} \approx 0$. Moreover, in the SM the $b \to s\gamma$ decay amplitude has a trivial weak phase (in the Wolfenstein parametrization) and the ratio $q/p = -e^{-2i\beta}$, which gives

$$\lambda_{\parallel} = \frac{q}{p} \frac{\bar{A}_L + \bar{A}_R}{A_L + A_R} \to e^{-2i\beta}, \qquad \lambda_{\perp} = \frac{q}{p} \frac{\bar{A}_R - \bar{A}_L}{A_R - A_L} \to -e^{-2i\beta}.$$
 (3.6)

The above results can be generalized to many extensions of the SM. When there is new CP conserving contribution to A_R , they are not modified. When there is new CP violating contribution to A_L the above results still hold, where one must replace 2β in both λ_{\parallel} and λ_{\perp} with the angle between the mixing and the decay amplitude. When there is new CP violating contribution to A_R , but no strong phase between the left and right handed amplitude, λ_{\parallel} and λ_{\perp} are still pure phase. However, the phase is not the same. Finally, when there is also a strong phase between the left and right handed amplitudes $|\lambda| \neq 1$. While we again assume the SM in the following discussion, our results hold also for the first two cases discuss above (with the general interpretation of 2β). It is clear how to generalize the results below also to more general cases.

Neglecting, as usual, the lifetime difference of the B_d mass eigenstates and assuming that $|\lambda| \approx 1$ one finds that each of the time-dependent CP asymmetries for the final states of linear polarization measure $\sin 2\beta$

$$a_{\parallel}(t) = \frac{|A_{\parallel}(t)|^2 - |\bar{A}_{\parallel}(t)|^2}{|A_{\parallel}(t)|^2 + |\bar{A}_{\parallel}(t)|^2} = -\text{Im}\lambda_{\parallel}\sin(\Delta mt) \to \sin 2\beta \sin(\Delta mt) \quad (SM)$$
(3.7)

$$a_{\perp}(t) = \frac{|A_{\perp}(t)|^2 - |\bar{A}_{\perp}(t)|^2}{|A_{\perp}(t)|^2 + |\bar{A}_{\perp}(t)|^2} = -\text{Im}\lambda_{\perp}\sin(\Delta mt) \to -\sin 2\beta \sin(\Delta mt) \quad (SM), \qquad (3.8)$$

whereas the corresponding CP asymmetry in the unpolarized rate is much suppressed [4]

$$A_{CP}(t) \equiv \frac{|A_{\parallel}(t)|^2 + |A_{\perp}(t)|^2 - |\bar{A}_{\parallel}(t)|^2 - |\bar{A}_{\perp}(t)|^2}{|A_{\parallel}(t)|^2 + |A_{\perp}(t)|^2 + |\bar{A}_{\parallel}(t)|^2 + |\bar{A}_{\perp}(t)|^2}$$

$$= \frac{|A_{\parallel}(0)|^2 - |A_{\perp}(0)|^2}{|A_{\parallel}(0)|^2 + |A_{\perp}(0)|^2} \sin(2\beta) \sin(\Delta mt).$$
(3.9)

It is interesting that much larger asymmetries are obtained for the coefficients of $\sin 2\phi$ and $\cos 2\phi$ in the angular dependence (2.15). Inserting the relations (3.1), (3.2) into (2.15) one finds a particularly simple time-dependence for the CP asymmetry of the $\cos 2\phi$ coefficient

$$\frac{\operatorname{Re}(A_R(t)A_L^*(t)) - \operatorname{Re}(\bar{A}_R(t)\bar{A}_L^*(t))}{|A_R(t)|^2 + |A_L(t)|^2 + |\bar{A}_R(t)|^2 + |\bar{A}_L(t)|^2 + |\bar{A}_L(t)|^2} = \frac{1}{2}\sin 2\beta \sin(\Delta mt).$$
(3.10)

Expressed in terms of the observed time-dependent angular distributions, this asymmetry can be written as

$$4 \frac{\langle \cos 2\phi \frac{d\Gamma(t)}{d\phi} \rangle - \langle \cos 2\phi \frac{d\bar{\Gamma}(t)}{d\phi} \rangle}{\langle \frac{d\Gamma(t)}{d\phi} \rangle + \langle \frac{d\bar{\Gamma}(t)}{d\phi} \rangle} = -\sin 2\beta \sin(\Delta mt).$$
 (3.11)

We denoted here $\langle f(\phi) \rangle = \int_0^{2\pi} d\phi f(\phi)$. Note that the result (3.11) does not depend on the smallness of the right-handed photon amplitude. On the other hand, the coefficient of $\sin 2\phi$ is more sensitive to the presence of a right-handed photon amplitude

$$\frac{\operatorname{Im}(A_{R}(t)A_{L}^{*}(t)) - \operatorname{Im}(\bar{A}_{R}(t)\bar{A}_{L}^{*}(t))}{|A_{R}(t)|^{2} + |A_{L}(t)|^{2} + |\bar{A}_{R}(t)|^{2} + |\bar{A}_{L}(t)|^{2}} =
- \frac{\operatorname{Re}(A_{\perp}A_{\parallel}^{*})}{|A_{\perp}|^{2} + |A_{\parallel}|^{2}} \cos 2\beta \sin(\Delta mt) - \frac{\operatorname{Im}(A_{\perp}A_{\parallel}^{*})}{|A_{\perp}|^{2} + |A_{\parallel}|^{2}} \cos(\Delta mt).$$
(3.12)

Assuming dominance by the left-handed amplitude in the SM (see Eq. (3.5)), one can use this asymmetry to extract $\cos 2\beta$

$$4 \frac{\langle \sin 2\phi \frac{d\Gamma(t)}{d\phi} \rangle - \langle \sin 2\phi \frac{d\bar{\Gamma}(t)}{d\phi} \rangle}{\langle \frac{d\Gamma(t)}{d\phi} \rangle + \langle \frac{d\bar{\Gamma}(t)}{d\phi} \rangle} = -\cos 2\beta \sin(\Delta mt). \tag{3.13}$$

While less clean theoretically than the determination of $\sin 2\beta$ from (3.11), this result is important because it can help us in resolving discrete ambiguities [29]. In the SM, once $\sin 2\beta$ is measured, we know β with no ambiguity from the bounds on the sides of the unitarity triangle. However, in the presence of physics beyond the SM the values of the "would be" β extracted from asymmetry measurements may not fall within its SM allowed range. Such new physics cannot be detected if the values of the asymmetry (i.e., $\sin 2\beta$) lie within the SM range. By measuring the sign of $\cos 2\beta$ we are sensitive to yet another kind of new physics: new CP violating contributions to the mixing amplitude.

IV. CONCLUSIONS

In this paper we argued that photon polarization information in exclusive weak radiative B decay can be used to probe new physics effects. The SM predicts that the photons emitted in $\bar{B}(B)$ decays are almost purely left (right) handed. By measuring the photon polarization we may find a signal for right-handed component that could only be generated by new physics. Moreover, since the linear polarization states are also CP eigenstates, the time-dependent CP asymmetries in $B^0(t)$ decays are clean. In the SM they measure $\sin 2\beta$;

by comparing to the CP asymmetries in $B \to \psi K_S$ decay, a possible new CP violating amplitude in the $b \to s\gamma$ decay (independent on its helicity) can be found.

We discuss two methods for determining the photon polarization by using the chain $B \to V \gamma \to V e^+ e^-$. In the first method, discussed also in [5,10], the photon is off-shell and we need to use the corresponding direct decay $B \to V e^+ e^-$ in the region where the dilepton invariant mass is close to the threshold since there photon exchange dominant the decay. The second method makes use of the Bethe-Heitler process where photon collide with matter and produce a lepton pair.

Two other methods were proposed in the past that are also sensitive to a right-handed component in the radiative decay amplitude. In [4] it was shown that a CP asymmetry in the radiative mode can be generated by right-handed amplitude. In [7] the $\Lambda_b \to \Lambda \gamma$ decay was studied and it was shown that the Λ polarization is sensitive to the right-handed operator \tilde{O}_7 .

We comment next on the experimental feasibility of these four methods. Experimentally, each method requires a different analysis, and thus at this stage we can only estimate their relative efficiency. As a benchmark we compare the efficiency of each of these measurement to the efficiency of the $B \to K^* \gamma$ rate measurement.

First, consider the method based on direct $B \to K^* e^+ e^-$ decay. Here, the major obstacle is statistics as the rate of the $B \to V e^+ e^-$ decay mode in the $q^2 < 1 \text{ GeV}^2$ region is smaller by about a factor of 10^{-2} compared to that of the radiative decay. However, electron pairs produced through virtual photons are most sensitive to the photon polarization (the corresponding efficiency parameter $\xi = 1$). Moreover, we could hope that the efficiency of the dileptonic mode will be higher than that of the radiative mode. The reconstruction of the dilepton pair emitted near the B decay point should be straightforward, as the corresponding tracks are expected to be well separated. In the lab frame, the maximum value of the opening angle between the e^+ and e^- momenta is $\tan \theta_{\rm max} = \sqrt{q^2 - (2m_e)^2}/|\vec{q}|$. For example, at $q_{\rm max}^2 = 0.5 \text{ GeV}^2$ the maximum value of this angle is $\theta_{\rm max} = 15^\circ$. Moreover, at hadron colliders, where the electron pair can be used for triggering, we could hope to get much higher efficiency in the semileptonic mode compared to the radiative mode. Thus, our rough estimates indicate that this decay has an efficiency of the order of few percent compared to the measurement of the $B \to K^* \gamma$ decay rate.

Next we look at the method using Bethe-Heitler lepton pairs. There are two major drawbacks here. First, the fraction of the photons that are converted is typically of the order of few percent, depending on the detailed matter content of the experimental apparatus. Second, the sensitivity to the photon polarization is not maximal ($\xi < 1$). On the positive side, we expect that the sensitivity to this mode will be higher compared to that of the $B \to K^*\gamma$ (where the γ is identified in the calorimeter) as the energy resolution of the lepton pair is higher and there is less background. The momenta of the conversion electrons produced in the inner layers of the detector are measured very well at CLEO (and the same is expected to be true at BaBar and Belle).

It seems that this kind of measurement is easier to be carried out at e^+e^- machines, as there is much less background. Yet, it is not impossible that also at hadron machines, where the statistics is much larger, this measurement can be done. We may conclude from our rough estimates that the efficiency for the Bethe-Heitler method is also at the few percent level.

The obvious advantage of the method proposed in [4] is that no polarization information is required. However, there are a few factors which offset this advantage. First, it is the fact that time-dependent measurements are necessary. Second, only neutral B decays can be used, while in the methods we suggested, also charged B (and Λ_b) can be used. Third, flavor tagging is needed. Last, the final state has to be a CP eigenstate, which gives further suppression of the rate through the chain $K^{*0} \to K_S \pi^0$. The combined effect of the above factors is a reduction in the efficiency of about a factor of a 100. We can conclude that this method seems somewhat disfavored compared to the methods we described.

Last we estimate the amount of data needed to carry out the suggestion of Ref. [7]. Since this method required Λ_b baryons, it is clear that this measurement can be done only at hadron machines. Moreover, Λ_b baryons are produced only about 10% of the time, and in general are harder to identify than B mesons. On the other hand it is relatively easy to collect the polarization information as the Λ decay provide it with high efficiency. Again, this very rough estimate suggests that if the radiative decays can be seen at hadron machines, the efficiency for the $\Lambda_b \to \Lambda \gamma$ decay with polarization information is at the few percent level compared to the $B \to K^* \gamma$ rate.

Finally, we comment on the feasibility of the CP asymmetries measurements discussed in Section III. It seems that these measurements are harder to perform since both polarization information and time dependent measurements are needed; thus they suffer from the problem of both our methods and the method of Ref. [4]. Yet, when we try to resolve discrete ambiguities only the sign of $\cos 2\beta$ is needed. Clearly, the sign of a specific quantity can be determined more easily than its magnitude, and requires less data. Therefore, we could still hope that the large numbers of B mesons expected to become available at the hadronic machines would make such measurements feasible.

Clearly, only a detailed experimental analysis can see which method is realistic. According to our estimates, it is possible that all the different analyses discussed above will be carried out.

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FIGURES

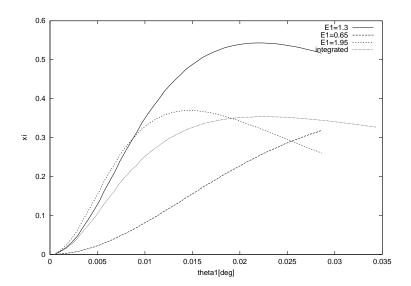


FIG. 1. The efficiency parameter ξ appearing in the angular distribution (2.20) of the Bethe-Heitler pairs for a polarized $E_{\gamma}=2.6$ GeV photon. The solid and dashed lines show the parameter ξ for given positron energies $E_1=1.3,0.65$ and 1.95 GeV, respectively. The dotted line shows the ξ parameter corresponding to an unobserved positron energy.